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# Phased Array Radar (PAR) Performance Considerations-

## *Impact of Errors*

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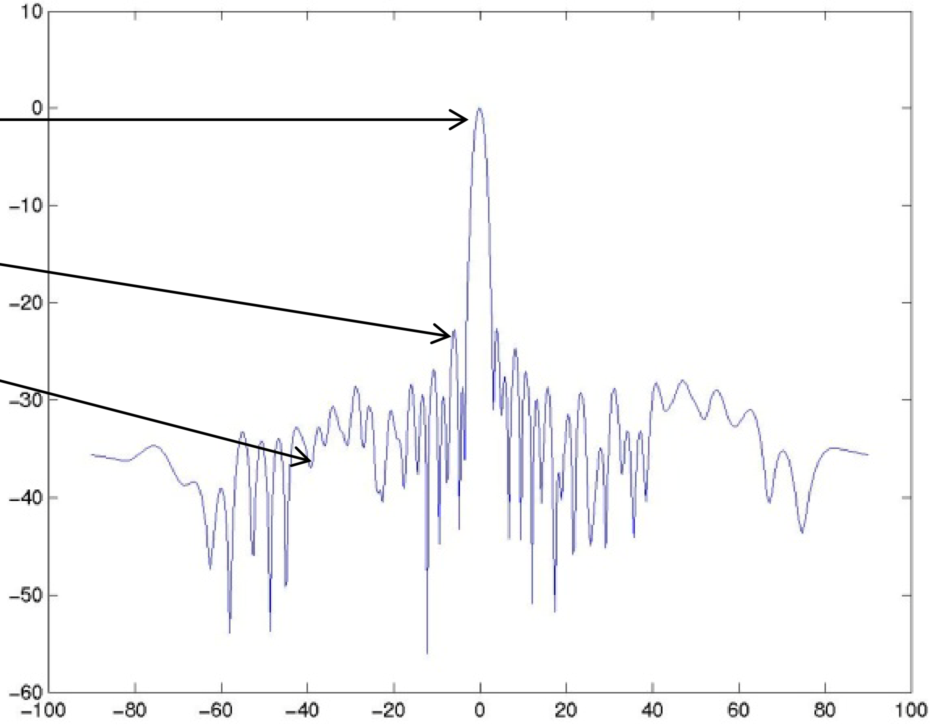
# Outline

- Beam characteristics
- Error definitions and sources
- Definition of the Correlation Interval
- Errors and Gain
- Errors and Sidelobes
- Errors and Pointing Accuracy (angle error)
- Errors and Polarization

# PAR Beam Characteristics



- Key Beam Characteristics
  - Main Beam
  - Peak Sidelobes
  - Average Sidelobes
- Main Beam (gain/sensitivity, tracking accuracy, coverage)
  - Beamwidth is approximately 57.3 l /Antenna Dimensions in Wavelengths (uniform illumination, rectangular aperture)
- Peak Sidelobes (interference, false targets)
  - Driven by amplitude taper across aperture
    - Uniform illumination = -13 dB
    - Tapered illuminations = XX dB
- Average Sidelobes (interference)
    - Driven by amplitude & phase errors

# Array Errors Definitions and Basics

- **What is meant by "Array Error"?**
- **Error = deviation from the ideal array**
  - **The Ideal array is a perfect implementation of the "designed" array which may still exhibit undesirable characteristics such as discretization effects, grating lobes, etc.**
  - **Ideal array performance is obtained from classical Array Theory**
- **e.g. If ideal element voltage = 1 then the actual voltage is $(1+ d)e^{jf}$**
- **There are three major effects of array errors which are of interest**
  - **Gain reduction**
    - **Energy is "stolen" from the main beam and distributed into the sidelobes**
  - **Sidelobe level (SLL) changes**
    - **Peak sidelobes**
    - **RMS or average sidelobes**
  - **Angle accuracy and pointing error**
    - **Errors add to noise contributions to degrade accuracy**

# Error Sources Identification

**Mechanical**

- Manufacturing tolerance
- Implementation error such as misalignment during assembly
- Distortion due to gravity or stress

**Electrical**

- Components
- VSWR
- T/R module
- Material tolerances
- Radome errors
- Surface tolerances
- Element outages
- Thermal noise

**Environmental**

- Temperature
- Rain/snow
- Wind
- Multi-path
- EMI

# Errors Are Categorized by Type

- Errors fall into two broad categories - Random and Correlated

- Random errors
  - Phase or amp errors which affect only one element (e.g. active array amplifier variations due to time, temperature, aging, etc.)
  - Failed elements
  - Element position
  - Non ideal phase shifter and attenuator (but not quantization)

- Correlated errors
  - Quantization of digital phase f shifter or attenuator
  - Errors affecting groups of elements e.g. subarray, beamformer, monopulse, digital receiver I/Q error etc.

- The mathematical treatment of errors depends on whether they are random or correlated and some errors can be treated either way depending on what performance criteria is of concern
  - e.g. Phase shifter quantization errors can be treated as random errors to calculate RMS SLL but peak SLL must be examined using classical array theory or a more detailed statistical analysis of the error distribution
- Effects due to random errors are given as ensemble averages

# Typical PAR Aperture Error Interval Levels

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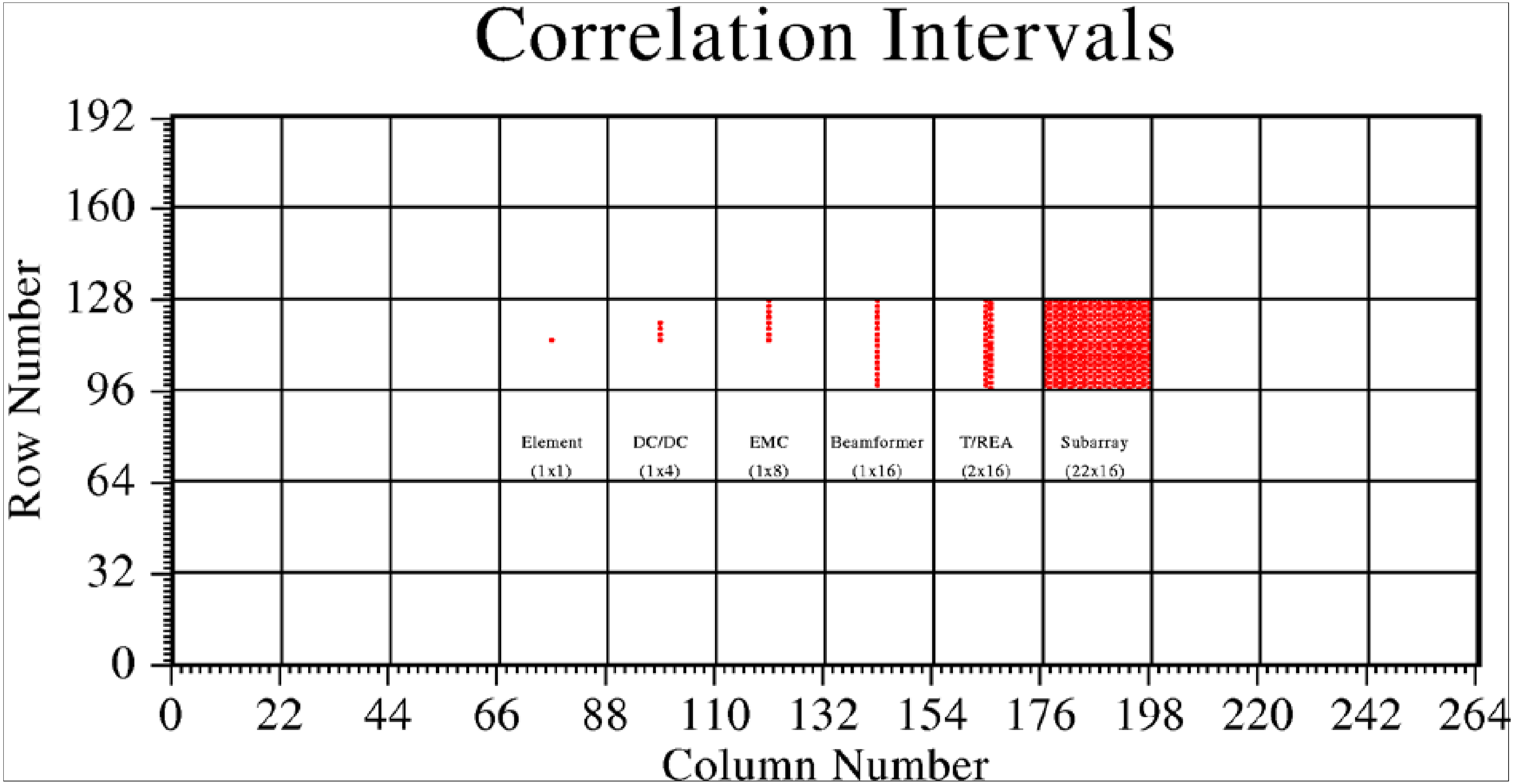
- **Correlation intervals are defined by the physical and electrical partitioning of the PAR and relative error impact**



T/REA – Transmit/Receive Element Assembly

EMC – Eight Module Controller

# PAR Gain Reduction Due to Errors

- **Gain is reduced by random and correlated errors**
  - **The treatment here is for errors which are statistically independent, of equal probability for all elements and possess zero mean value**

**Gain at Peak of scanned Main Beam**

$$G(q) = h_O h_I h_S G_e(q)\ 4p A/ l^{2}$$

$G_e(q)$ = Element pattern = $(1-½G^{2}·(q)½)$ cos(q)

G(q) = active reflection coefficient of the array

q = array scan angle

l = wavelength

$h_O$ = ohmic losses of radiating element & others

$h_I$ = illumination eff = $(SW_i)^2 / N / (SW_i^2)$

N = total number of array elements

$W_i$ = array weighting of the i-th element

**Deterministic Parameters**

**Statistical Parameter**

$h_S$ = similarity loss due to array errors

Similarity loss $h_S = P / (1+d^2+f^2)$

where P is survival probability of radiating elements = $1 - N_F/N$ and $N_F$ = number of failed elements

d = RMS amplitude error in Volts/Volt ~ RMS amplitude error in dB / 8.686

f = RMS phase error in radians

$d^2 = d_E^2 + d_S^2 + d_B^2$ where E,S, and B stand for contribution from element, subarray, beamformer

$d_E^2 = d_{EQ}^2 + d_{ER}^2$ where Q and R represent quantization and RMS error, respectively.

$d_{EQ}$ = RMS attenuator quantization error = LSB / (2Ö3) where LSB = least significant bit of attenuator in Volts/Volt

$f_{EQ}$ = RMS phase shifter quantization error = 2p / ($2^m$Ö12) where m = number of phase shifter bits

$f_{ER}$ = RMS position phase error = (2p / l )* RMS positional error

# Gain Reduction Example

- Consider an array with 5 bit phase shifter, 4 bit/16dB attenuators, subdivided into subarrays with RMS feed errors of .5 dB and 3 deg at subarray level, RMS element position tolerance of .02cm, element RMS random errors of .2 dB and 1deg, freq = 10GHz, .1 dB and 2deg RMS monopulse errors, 2% failed elements

  Similarity loss $h_S = P / (1+d^2+f^2)$

  $f_{EQ} = 2p / (2^5Ö12 = .057$ (phase shifter quantization error)

  $f_{ER1} = .02*(2p / 3) = .042$ (position error)

  $f_{ER2} = (p/180) = .017$ (random phase error)

  $f_E^2 = f_{EQ}^2 + f_{ER1}^2 + f_{ER2}^2 = .057^2 + .042^2 + .017^2 = .005$ (total element phase error)

  $f^2 = f_E^2 + f_S^2 + f_B^2 = .005 + (3p/180)^2 + (2p/180)^2 = .009$ (total phase error)

  $d_{EQ} = LSB / (2Ö3) = 1dB/ (2Ö3) = .289\ dB \sim .033$ volts/volt (attenuator quant error)

  $d_E^2 = d_{EQ}^2 + d_{ER}^2 = .033^2 + (.2dB/8.686)^2 = .002$ (total element amplitude error)

  $d^2 = d_E^2 + d_S^2 + d_B^2 = .002 + (.5dB/8.686)^2 + (.1dB/8.686)^2 = .005$ (total amplitude error)

  $h_S = .98/(1+.009+.005) = .966 = -.15dB$ (total gain loss due to errors)

  - The level of errors needed to get low sidelobes results in small gain loss
  - Note this result does not depend on array size, number of elements or taper

# Sidelobe Errors

- **Sidelobe contributions from random errors appear in the far field with the directivity of a single element**
  - **= p for l /2 element spacing and is often referenced to that level in the literature - the equations given here are referenced to main beam peak when beam is scanned to broadside**
- **The RMS average sidelobe level due to random errors is equal to the energy lost from the main beam, normalized to the beam peak**
- **Sidelobe contributions from correlated errors at the element, subarray and beamformer level are predicted by breaking the array into subarrays which each have a constant value of the error**
  - **These errors can be large even for well designed arrays under certain circumstances such as wide instantaneous bandwidth operation**

$$\mathrm{SLL}_{\mathrm{RMS}} = S(d_i^2+f_i^2)/hN_i + (1-P)/(hNP)$$

**where i = E, S and B for element, subarray and beamformer**

**Note: this is the average SLL at a point in the pattern where the ideal array has a null, it is the RMS SLL due to the errors alone**

# Sidelobe Error Example

- Same array as previous example - assume the array is circular, 1000 elements, dx=dy= l /2, 30 dB/ n=8 Taylor taper, subdivided into 4 subarrays of 250 elements each

h = .884 taper loss or aperture efficiency can be found in tables or calculated from the ideal (error free) array distribution

$SLL_{RMS} = S(d_i^2+f_i^2)/N_i + (1-P)/(hNP)$

$= (.002 +.005)/(.884*1000)$ (element level errors = 7.92x10e-6)

$+ ((.5/8.686)^2 + (3p/180)^2)/(.884*4)$ (subarray level errors = 1.713x10e-3)

$+ ((.1dB/8.686)^2 + (2p/180)^2)/(.884*4)$ (monopulse level errors = 3.82x10e-4)

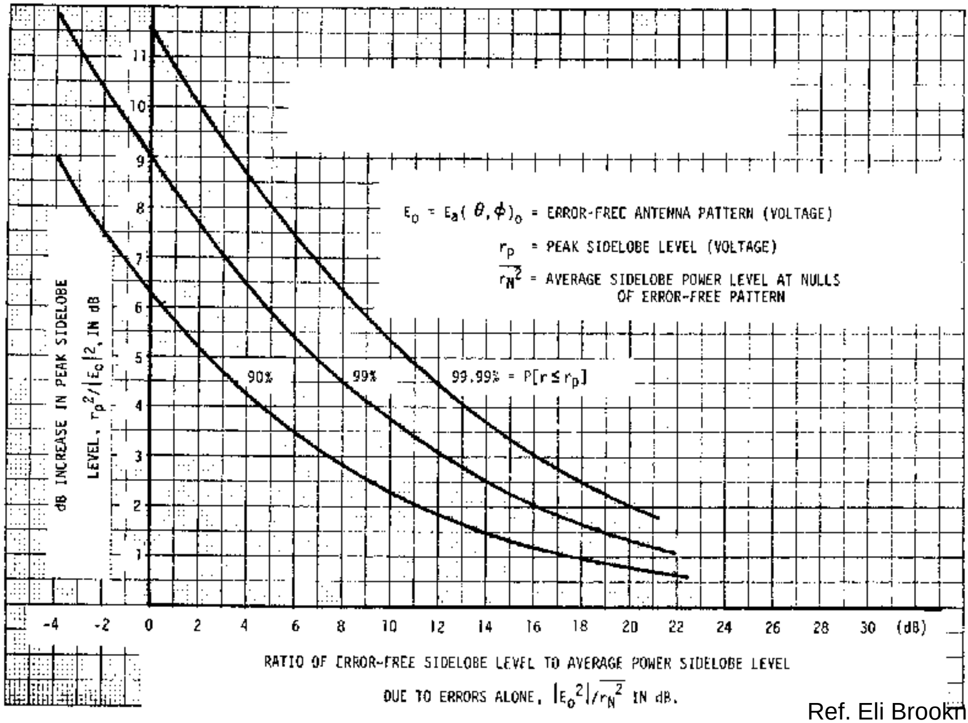
$+ (1-.98)/(.884*1000*.98)$ (failed element contribution = 2.61x10e-5)

$SLL_{RMS} = -26.7$ dB

- The subarray contribution is the principal cause of high RMS sidelobes
- These results are for broadside beam pointing, as the array is scanned the peak gain drops so the relative SLL worsens. Correct for this with the embedded element pattern gain at the scan and sidelobe locations.

# Peak Sidelobe Degradation

- The peak SLL can be estimated statistically by combining the Rayleigh distributed random sidelobes with the ideal array sidelobes.
- The chart gives the peak sidelobe degradation as a function of the error SLL / ideal SLL ratio with probability level as a parameter



Ref. Eli Brookner, “Practical Phased Array Antenna Systems”

- e.g. an array has ideal SLL of -30dB and RMS SLL of -25dB, then the peak SLL will be <26.2dB 90% of the time and <24dB 99% of the time

# Angle Error

**RMS Angle error $\sigma^2 = \sigma_B^2 + \sigma_R^2 + \sigma_T^2$**

where B, R, and T stand for contribution from common bias, random error, and thermal noise, respectively.

$\sigma_R = \theta_3 \phi_{eff} / \eta K$

$\phi_{eff}$ is effective phase error due to array components, $\phi_{eff}^2 = \phi_E^2/N_E + \phi_S^2/N_S + \phi_B^2/N_B$

where $N_E$, $N_S$, $N_B$ are numbers of element, subarray and beamformer, respectively

K = angle sensitivity of monopulse pattern

$= \theta_3(2\pi/\lambda)(\Sigma X_i W_{Di}) / \sqrt{N \Sigma W_{Di}^2}$

$W_{Di}$ = difference weighting of the ith element

$X_i$ = X-coordinate of the i-th element

$\sigma_T$ = angle error due to thermal noise

$= (\theta_3/K_m) / \sqrt{2\ SNR}$ where SNR = signal to noise ratio of array

$K_m$ = monopulse slope of monopulse beam

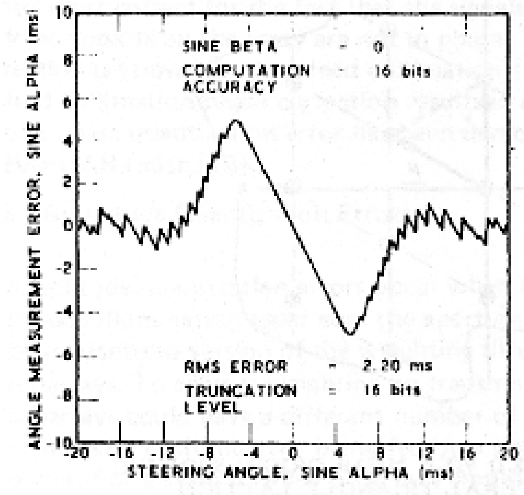
$= K_s \sqrt{T_S/T_D}$ where $T_S$, $T_D$ = thermal noise in sum and delta channels

$Ks = \partial(\Delta/\Sigma) / \partial(\theta/\theta_3)|_{\theta=0}$

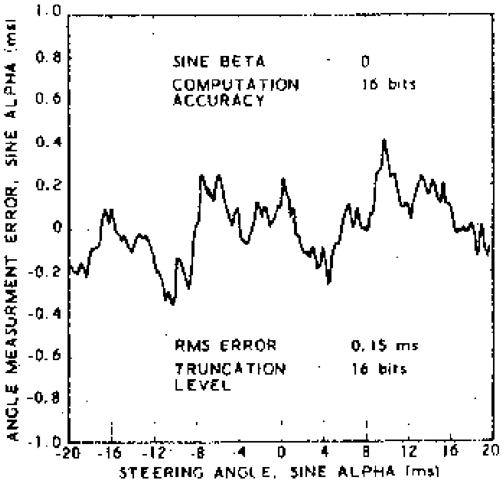
# Angle Error Due to Phase Shifter Quantization

- **Peak pointing error due to periodic phase error arising from digital phase shift can cause much greater angle error than that due to random errors**
- **Dsin(q) = l /Nd2p where N = number of elements, d = element spacing and p = number of bits**
  - **e.g. 50x50 array, d = l /2, p = 3 bits yields 5ms angle error***
- **This error can be reduced by randomizing the phase bits. Insert additional phase length in element feeds and compensate with phase bit settings, dither the phase settings or beam position from pulse to pulse, etc.**
- **Example is shown for 26.4 l diameter, 1891 element circular array**

**Non Randomized Phase Settings**



**Randomized Phase Settings**



***Note ms = millisines ~ angle in milliradians for small angles**

Ref. Eli Brookner, “Practical Phased Array Antenna Systems”

# The Importance of Polarization

- Polarization allows us to determine certain geometric features of the scattering object
  - Distinguishes various phases of precipitation e.g. ice, hail and liquid
  - Quantifies rain, snow and hailfall rates
- Polarization quality and usefulness is determine by many factors
  - PAR mutual coupling effects between radiating elements as a function of frequency and beam pointing angle
  - Radiating element mismatch
  - Impact of the radome and environments

# Polarization Modes

- In order to acquire the dual-polarization signature of the targets, two polarization modes are usually implemented, one with sequential polarization transmission (alternate mode, or VH), the other with simultaneous polarization transmission (simultaneous mode, or VHS).
- VH: radar measures both co-polar radar parameters and cross-polar radar parameters.
  - The cross-polar component of the echoes are very weak, typically < -25 dB for rain.
- VHS: radar measures the co-polar radar parameters.

# Evaluation of the Cross-polarization Coupling

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- How good does the antenna/system need to be?

Define as integrated CPL (<u>C</u>ross-<u>P</u>olarization <u>L</u>evel)

| Retrieval Biases | Accuracy Constraints | Isolation Requirements | | Alignment Requirement (VHS mode) | | | |
|---|---|---|---|---|---|---|---|
| | | VH mode | VHS mode | $F_{dp}$<30° | $F_{dp}$<60° | $F_{dp}$<100° | $F_{dp}$<180° |
| $DZ_h$ | < 0.5 dB | - | CPL<-30dB | 7.2° | 4.5° | 2.9° | 1.8° |
| | < 1.0 dB | - | CPL<-25dB | - | - | 5.7° | 3.2° |
| $DZ_{dr}$ | < 0.2 dB | CPL<-20dB | CPL<-44dB | 4.5° | 1.3° | 0.57° | 0.36° |
| | < 0.3 dB | CPL<-18dB | CPL<-42dB | 6.4° | 1.8° | 0.81° | 0.46° |
| $DK_{dp}$ | < 10% | - | CPL<-28dB | | | | |

$\Delta Z_{dr}$ (differential reflectivity-used for droplet characterization) is very sensitive to the polarization deviation for VH mode, but all biases are more sensitive for VHS mode

- What happens when the cross-polarization isolation is low?
  – Need to consider effects of non-diagonal propagation which generates uncertainties

# Radome Effects on Polarization

- Curved and flat radomes affect the polarization performance of apertures
- Heavy rain will cause a beam broadening loss due to phase errors
- A flat radome in front of a planar array will degrade the polarization as follows

| Frequency | Cross-polarization level (dB) for Dry radome | Cross-polarization level (dB) for Hydrophobic surface | Cross-polarization level (dB) for Non-hydrophobic surface | Comments |
|---|---|---|---|---|
| S band | -38.5 | -30 | -17 | 60 deg incident angle at the diagonal plane, 20 mm/hr rain rate |

# References

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